SYNOPTIC: Optimal Controls for Out-of-Plane Motion about the Translunar Libration Point,

T. A. Heppenheimer, University of Michigan, *Journal of Spacecraft and Rockets*, Vol. 7, No. 9, pp. 1088–1092.

Lunar and Planetary Trajectories; Spacecraft Mission Studies and Economics

Theme

The problem is to control the period of the out-of-plane motion about the lunar L_2 point, using impulsive thrust, to prevent a libration-point satellite from being occulted by the Moon. The operational use of the control is also discussed.

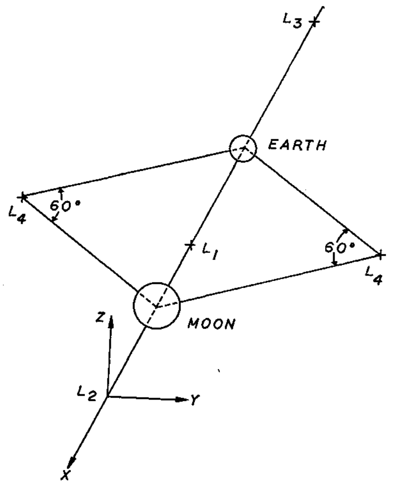

Fig. 1 The five libration points in the Earth-moon system, and the coordinate system used in this paper.

Content

A restricted three-body problem model is used to describe the motion, and the equations of motion are linearized about the lunar L_2 point (Fig. 1). The out-of-plane or z motion is simple harmonic with period 15.2963 days; in-plane (x–y) motion, when stabilized, has period 14.6683 days. A fuel-optimal period control is synthesized using algebraic minimization, with the z motion being studied in the phase plane. The amplitude A_z of the z oscillation is constrained below ($A_z \geq A_{z0}$), and the resulting control is as follows.

There exists a family of local optima, the nth member of which is called Mode n. Mode n involves one thrust every $n/2$ oscillations, the thrust occurring ($3.8241–0.1570n$) days subsequent to passage through the x-y plane. Let A_z be in units of 10^3 km; then the ΔV per burn is given by $\Delta V = 31.14 \sin(0.06449n)$. Let z_0 define the radius of a zone of occultation; then $A_z = A_{z0} = z_0/\cos(0.06449n)$.

It thus is possible to tradeoff fuel consumption against interval-between-thrusts. It also is possible that for operational reasons (e.g., to avoid interrupting use of the satellite) a scheduled thrust may be advanced or delayed. Delays ($+\delta$) always increase both amplitude and ΔV; advances ($-\delta$) decrease A_z and thus may produce occultation (see Figs. 2 and 3). For a sufficiently long delay, it is advantageous to cancel the scheduled burn in Mode n and make the next burn in Mode ($n + 1$), $\frac{1}{2}$ oscillation later. The associated penalty may then be bounded above and is given by the dashed horizontal lines in Fig. 3; LUB = least upper bound, corresponding to burns at $z = z_0$. The fuel penalties are normalized with respect to the ΔV per burn for Mode 1, which is taken as unity. For example, at $\delta = 0$ in Fig. 3, the interval between thrusts can be increased tenfold for a 16.5% fuel penalty.

It is also possible that there will be extended intervals during which use of the satellite will be so limited that it

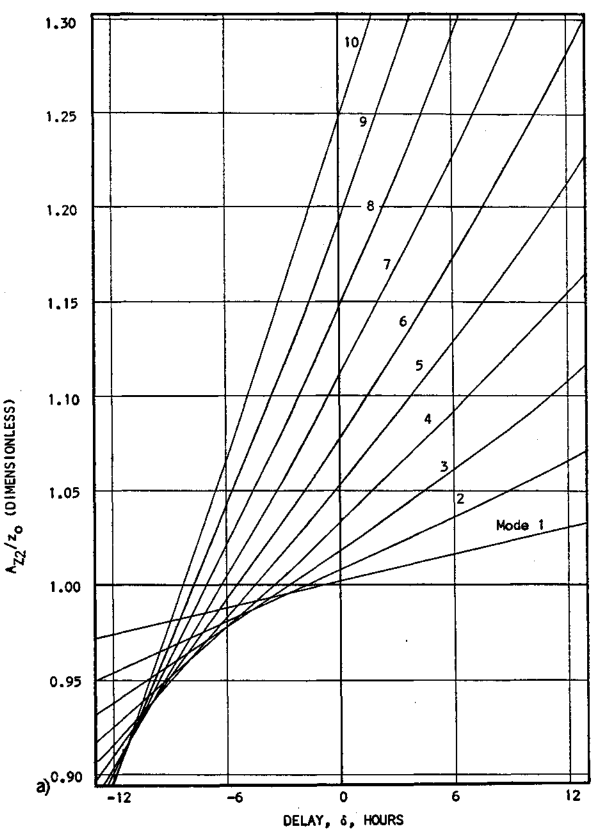

Fig. 2 Effect on fuel economy and amplitude A_{z_2} of period-control thrusts made at nonoptimal times; postburn amplitude A_{z_2}; z_0 = const.

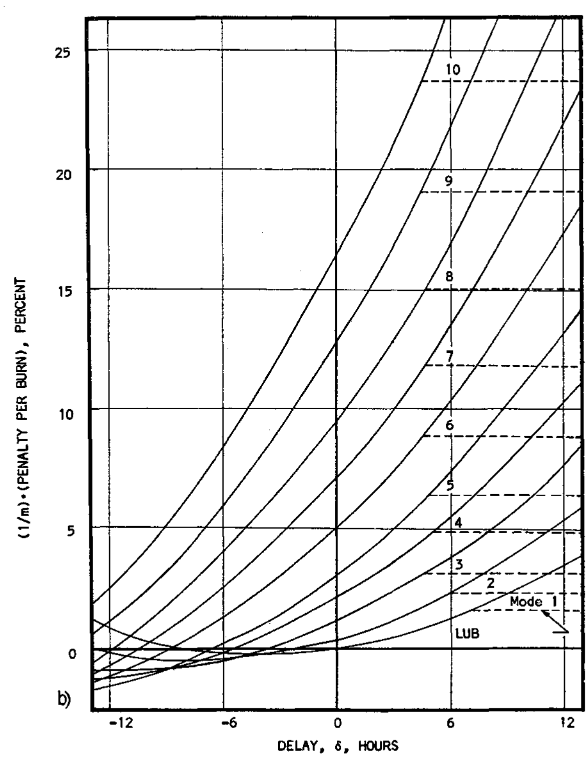

Fig. 3 Effect on fuel economy and amplitude A_{z_2} of period-control thrusts made at nonoptimal times; fuel penalty with respect to the globally optimal Mode 1.

would be acceptable for occultation to occur. During this interval, the period control could be powered down (i.e., not used), and fuel would be saved; however, when the satellite was to be used again, a large ΔV would be required to achieve transfer from the occulting orbit to an occultation-free orbit. Nevertheless, it is found that power-down produces a net fuel saving whenever the time of limited use exceeds the nominal interval-between-thrusts.

Relation to Literature

These results are in conflict with a period control proposed by Farquhar ("Lunar Communications with Libration Point Satellites," *Journal of Spacecraft and Rockets*, Vol. 4, No. 10, October 1967, pp. 1383–1384). Farquhar's solution, however, involves substantially higher cost and in addition represents a single optimum and not a family of local optima. Moreover, his solution is objectionable on theoretical grounds since its characteristics are strongly dependent upon the small parameter e, the lunar eccentricity, and his solution actually fails to exist for $e = 0$. The preceding solution, though computed for the case $e = 0$, requires only slight modification to accommodate the actual case of $e \ll 1$ (i.e., A_z must be increased). In this light, Farquhar's requirement that $e > 0$ appears artifical.

Optimal Controls for Out-of-Plane Motion about the Translunar Libration Point

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For a spacecraft in stabilized motion about the lunar L_2 point, the periods of the in-plane and out-of-plane oscillations must be synchronized to prevent occultation of the spacecraft by the moon. Phase-plane methods are used to construct a family of locally fuel-optimal out-of-plane period controls, implemented with impulsive thrust. A tradeoff between fuel penalty and interval-between-thrusts is derived, and it is shown that the effects of anomalous thrust vectors may be easily dealt with. For operational reasons, impulses may have to be made at nonoptimum times; it is shown that the associated fuel penalty can be bounded above. During times of limited use of the satellite it may be possible to power-down the period control; it is shown how power-up may be accomplished so that there is a net saving in fuel. Results are plotted in a form useful for mission planning and operation.

Introduction

A TOPIC of some current interest is the establishment and control of a satellite in the vicinity of the translunar libration point or L_2 point, which is one of the five points of equilibrium in the Earth-moon gravitational field. Such a satellite could be used for radio astronomy, communications with the lunar farside, solar-wind studies, and the like. Among the most detailed studies of the astrodynamics problems associated with such satellites have been those of Farquhar[1] and of General Electric.[2]

To the accuracy required by a preliminary study, motion in the vicinity of the L_2 point may be studied by means of a restricted three-body problem model wherein the effects of lunar eccentricity ($e = 0.05490$) are implicitly neglected, as are the perturbing effects of fourth bodies (e.g., the Sun). Near the L_2 point the governing equations of motion may be linearized. Following the derivation given by Szebehely,[3] these linearized equations of motion take the form

$$\ddot{x} - 2\dot{y} - 7.38085x = 0 \qquad (1)$$

$$\ddot{y} + 2\dot{x} + 2.19042y = 0 \qquad (2)$$

$$\ddot{z} + 3.19042z = 0 \qquad (3)$$

where x,y,z form a right-handed system as in Fig. 1, and time t is normalized with respect to the lunar mean motion; unit time is 4.34838 days.

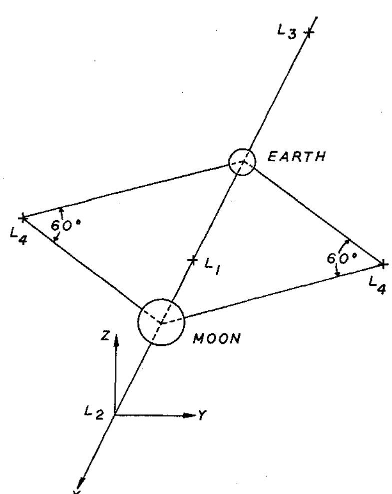

Fig. 1 The five libration points in the Earth-moon system, and the coordinate system used in this paper.

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